Works of T. Coquand in constructive algebra

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Abstract

Here is a list of references w.r.t. works of T. Coquand in constructive algebra.

1 Distributive lattices, algebraization of logic

[Stone, 1937, Topological representations of distributive lattices and Brouwerian logics]
[Hochster, 1969, Prime ideal structure in commutative rings] Hochster does not cite Stone 37.
[Joyal, 1971, Spectral spaces and distributive lattices]
[Johnstone, 1982, Stone spaces] Very important book

[Cederquist and Coquand, 2000, Entailment relations and distributive lattices]
(They cite Johstone 1982)
Summary. To any entailment relation [Sco74] we associate a distributive lattice. We use this to give a construction of the product of lattices over an arbitrary index set, of the Vietoris construction, of the embedding of a distributive lattice in a boolean algebra, and to give a logical description of some spaces associated to mathematical structures.

[Coquand, 2005a, A completeness proof for geometrical logic]
[Coquand, 2005b, About Stone's notion of spectrum]
[Bezem and Coquand, 2005, Automating coherent logic]
[Bezem and Coquand, 2019, Skolem's Theorem in Coherent Logic]
[Bezem and Coquand, 2022, Loop-checking and the uniform word problem for join-semilattices with an inflationary endomorphism]

2 Hilbert's programme for constructive algebra

2.1 General presentation

[Coquand, 1997, Computational content of classical logic] (a course given in 1995)
[Berardi, Bezem, and Coquand, 1998, On the computational content of the axiom of choice]
[Coste, Lombardi, and Roy, 2001, Dynamical method in algebra: effective Nullstellensätze]
[Coquand, 2003, Dynamical methods in algebra]
[Coquand and Lombardi, 2006, A logical approach to abstract algebra]
[Coquand and Lombardi, 2013, Constructive algebra]

2.2 Some remarquable results

[Coquand, 2006, On seminormality]
Summary. We give an elementary and essentially self-contained proof that a reduced ring R is seminormal if and only if the canonical map $\mathrm{Pic}(R) \to \mathrm{Pic}(R[X])$ is an isomorphism, a theorem due to Swan [R. Swan, On seminormality, J. Algebra 67 (1980) 210–229], generalizing some previous results of Traverso [C. Traverso, Seminormality and the Picard group, Ann. Scuola Norm. Sup. Pisa 24 (1970) 585–595].

[Coquand, 2007, A refinement of Forster's theorem]
Summary. The book [H. Bass Algebraic K-theory. W. A. Benjamin, Inc., New York-Amsterdam 1968] contains a refinement of Serre's Splitting-Off Theorem, which actually applies also to Forster's Theorem. We give an elementary proof of this refinement.

[Alonso García, Coquand, and Lombardi, 2014, Revisiting Zariski main theorem from a constructive point of view]
Summary. This paper deals with the Peskine version of Zariski Main Theorem published in 1965 and discusses some applications. It is written in the style of Bishop's constructive mathematics. Being constructive, each proof in this paper can be interpreted as an algorithm for constructing explicitly the conclusion from the hypothesis. The main non-constructive argument in the proof of Peskine is the use of minimal prime ideals. Essentially we substitute this point by two dynamical arguments; one about gcd's, using subresultants, and another using our notion of strong transcendence. In particular we obtain algorithmic versions for the Multivariate Hensel Lemma and the structure theorem of quasi-finite algebras.
Strong transcendence: Let D be a C-algebra and $x \in D$. We say that x is strongly transcendent over C in D if for all $u \in D$ and $c_0, \dots, c_k \in C$ such that $u(c_0+\cdots+c_k x^k) = 0$, we have $uc_0 = \cdots = uc_k = 0$.

2.3 Krull dimension

[Joyal, 1976, Les théoremes de Chevalley-Tarski et remarques sur l'algèbre constructive]
[Español, 1978, Dimensión en álgebra constructiva]
[Lombardi, 2002, Dimension de Krull, Nullstellensätze et évaluation dynamique]
[Coquand and Lombardi, 2003, Hidden constructions in abstract algebra: Krull dimension of distributive lattices and commutative rings]

[Coquand, 2004, Sur un théorème de Kronecker concernant les variétés algébriques]
Summary. A classical result of Kronecker, stated at the end of the Section 10 of Kronecker (J. Reine Angew. Math. 92 (1882) 1–123), is that any radical of a finitely generated ideal in a polynomial ring of n variables is the radical of an ideal generated by $n+1$ elements. We give a constructive and elementary proof of a generalisation presented in (Heitmann, Generating non-Noetherian modules efficiently, 1984): in a ring of Krull dimension n a radical of a finitely generated ideal is the radical of an ideal generated by $n+1$ elements

[Coquand, Lombardi, and Quitté, 2004, Generating non-Noetherian modules constructively] Summary. In [6], (Generating non-Noetherian modules efficiently. Michigan Math. J. 31 (2), 167–180 (1984)), Heitmann gives a proof of a Basic Element Theorem, which has as corollaries some versions of the "Splitting-off" theorem of Serre and the Forster-Swan theorem in a non-Noetherian setting. We give elementary and constructive proofs of such results. We introduce also a new notion of dimension for rings, which is only implicit in [6] and we present a generalisation of the Forster-Swan theorem, answering a question left open in [6].

[Coquand, Lombardi, and Roy, 2005a, An elementary characterization of Krull dimension]
Summary. We give an elementary characterisation of Krull dimension for distributive lattices and commutative rings. This follows the following geometrical intuition: an algebraic variety is of dimension $\leqslant k$ if and only if each subvariety has a boundary of dimension $< k$. Since our results hold for distributive lattices, they hold, by Stone duality, for any spectral spaces.

[Coquand and Lombardi, 2005, A short proof for the Krull dimension of a polynomial ring]
[Coquand, Ducos, Lombardi, and Quitté, 2009a, Constructive Krull dimension. I. Integral extensions]
[Coquand and Lombardi, 2018, Constructions cachées en algèbre abstraite. Dimension de Krull, Going Up, Going Down]
[Coquand, Lombardi, and Quitté, 2020b, Dimension de Heitmann des treillis distributifs et des anneaux commutatifs]

2.4 Constructive algebraic closure

[Cohen and Coquand, 2013, A constructive version of Laplace's proof on the existence of complex roots]
[Coquand and Lombardi, 2008, A note on the axiomatization of real numbers]
[Mannaa and Coquand, 2013, Dynamic Newton-Puiseux theorem]
[Coquand, Lombardi, and Neuwirth, 2021, Constructive basic theory of central simple algebras]

2.5 Finite free resolutions

[Coquand and Quitté, 2012, Constructive finite free resolutions]
Summary. Northcott's book Finite Free Resolutions (1976), as well as the paper (J. Reine Angew. Math. 262/263:205–219, 1973), present some key results of Buchsbaum and Eisenbud (J. Algebra 25:259–268, 1973; Adv. Math. 12: 84–139, 1974) both in a simplified way and without Noetherian hypotheses, using the notion of latent nonzero divisor introduced by Hochster. The goal of this paper is to simplify further the proofs of these results, which become now elementary in a logical sense (no use of prime ideals, or minimal prime ideals) and, we hope, more perspicuous. Some formulations are new and more general than in the classical results.

[Coquand and Tête, 2018, An elementary proof of Wiebe's theorem]
Summary. This note has three contributions. The first one is to give elementary proofs about finite free resolution using only the notion of latent regular element (regular element which may be obtained after addition of indeterminates) and no notion from homological algebra. The second contribution is to use this to provide a simple proof of Wiebe's Theorem, a fundamental result in the theory of resultants. Finally the third contribution is to explain how to "automatically" prove results or obtain counter-examples about regular sequences of a given length. We obtain in this way a new example of a local ring with a regular sequence b_1, b_2 such that b_2, b_1 is not regular.

[Coquand, Lombardi, Quitté, and Tête, 2020a, Résolutions libres finies. Méthodes constructives]

2.6 Valuation theory

[Coquand and Persson, 2001, Valuations and Dedekind's Prague theorem]

[Coquand, 2009, Space of valuations]
[Coquand and Lombardi, 2016a, Anneaux à diviseurs et anneaux de Krull (une approche constructive)]

2.7 Grothendieck schemes

[Coquand, Lombardi, and Schuster, 2007, The projective spectrum as a distributive lattice]
[Coquand, Lombardi, and Schuster, 2009b, Spectral schemes as ringed lattices]

2.8 Lorenzen's work in algebra

[Coquand, Lombardi, and Neuwirth, 2021, Regular entailment relations]
[Coquand, Lombardi, and Neuwirth, 2019, Lattice-ordered groups generated by an ordered group and regular systems of ideals]
[Coquand, 2021, Lorenzen and constructive mathematics]

2.9 Miscellanous

[Coquand and Persson, 1999, Gröbner bases in type theory]
[Coquand and Lombardi, 2001, Krull's Principal Ideal Theorem]
[Coquand, Lombardi, and Schuster, 2005b, A nilregular element property]
[Coquand, Lombardi, and Quitté, 2010, Curves and coherent Prüfer rings]
[Coquand and Lombardi, 2016b, Some remarks about normal rings]
[Coquand and Spitters, 2012, A constructive proof of Simpson's rule]

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